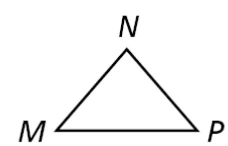
Attendance Problems.

1. Find the measure of exterior $\measuredangle DBA$ of $\triangle BCD$, if $m\measuredangle DBC = 30°, m\measuredangle C = 70°,$ and $m\measuredangle D = 80°$.

2. What is the complement of an angle with measure 17°?

3. How many lines can be drawn through N parallel to $\overline{MP}$? Explain why.



- I can find the measures of interior and exterior angles of triangles.
- I can apply theorems about the interior and exterior angles of triangles.

Vocabulary		
auxiliary line	corollary	interior
exterior	interior angle	exterior angle
remote interior angle		

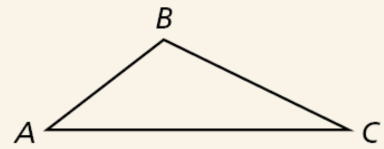
CC.9-12.G.CO.10 Prove theorems about triangles.

Q: How many feet are in a yard?
A: It depends on how many people are in the yard!

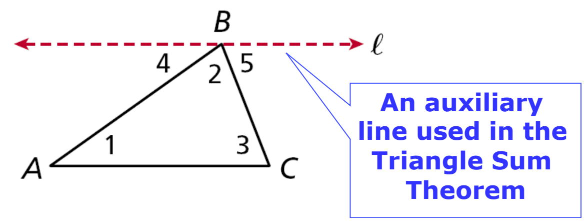
Theorem 4-2-1 **Triangle Sum Theorem**

The sum of the angle measures of a triangle is 180°.

$m\angle A + m\angle B + m\angle C = 180°$



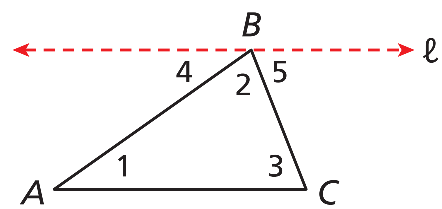
An **auxiliary line** is a line that is added to a figure to aid in a proof.



Triangle Sum Theorem

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Proof:

Draw $\ell \parallel \overline{AC}$ through B.
Parallel Post.

$\angle 1 \cong \angle 4$
Alt. Int. ∠s Thm.

$\angle 3 \cong \angle 5$
Alt. Int. ∠s Thm.

$m\angle 1 = m\angle 4$
Def. of ≅ ∠s

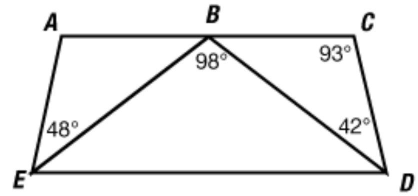
$m\angle 3 = m\angle 5$
Def. of ≅ ∠s

$m\angle 4 + m\angle 2 + m\angle 5 = 180°$
∠ Add. Post. & def. of straight ∠

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$
Subst.

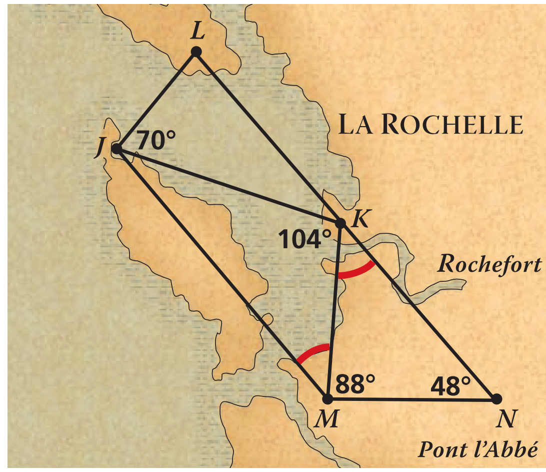
<u>Video Example 1</u>: Use the diagram to find the following measures.

A. $m\measuredangle CBD$. **B.** $m\measuredangle EAB$.



1 Surveying Application

The map of France commonly used in the 1600s was significantly revised as a result of a triangulation land survey. The diagram shows part of the survey map. Use the diagram to find the indicated angle measures.



A $m\angle NKM$

$m\angle KMN + m\angle MNK + m\angle NKM = 180°$	*△ Sum Thm.*
$88 + 48 + m\angle NKM = 180$	*Substitute 88 for m∠KMN and 48 for m∠MNK.*
$136 + m\angle NKM = 180$	*Simplify.*
$m\angle NKM = 44°$	*Subtract 136 from both sides.*

B $m\angle JLK$

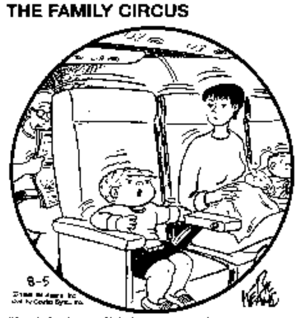
Step 1 Find $m\angle JKL$.

$m\angle NKM + m\angle MKJ + m\angle JKL = 180°$	*Lin. Pair Thm. & ∠ Add. Post.*
$44 + 104 + m\angle JKL = 180$	*Substitute 44 for m∠NKM and 104 for m∠MKJ.*
$148 + m\angle JKL = 180$	*Simplify.*
$m\angle JKL = 32°$	*Subtract 148 from both sides.*

Step 2 Use substitution and then solve for $m\angle JLK$.

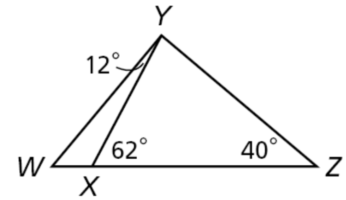
$m\angle JLK + m\angle JKL + m\angle KJL = 180°$	*△ Sum Thm.*
$m\angle JLK + 32 + 70 = 180$	*Substitute 32 for m∠JKL and 70 for m∠KJL.*
$m\angle JLK + 102 = 180$	*Simplify.*
$m\angle JLK = 78°$	*Subtract 102 from both sides.*

THE FAMILY CIRCUS



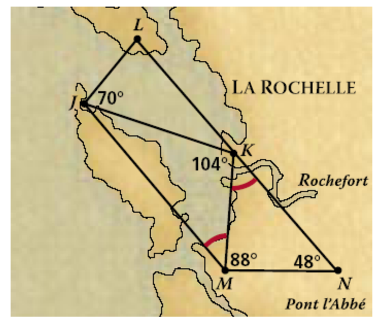
"I wish they didn't turn on that seatbelt sign so much! Every time they do, it gets bumpy."

Example 1. **After an accident, the positions of cars are measured by law enforcement to investigate the collision. Use the diagram drawn from the information collected to find the angle measures.**



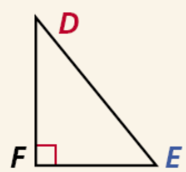
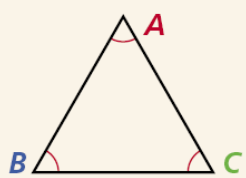
A. $m\measuredangle XYZ$. **B.** $m\measuredangle YWZ$.

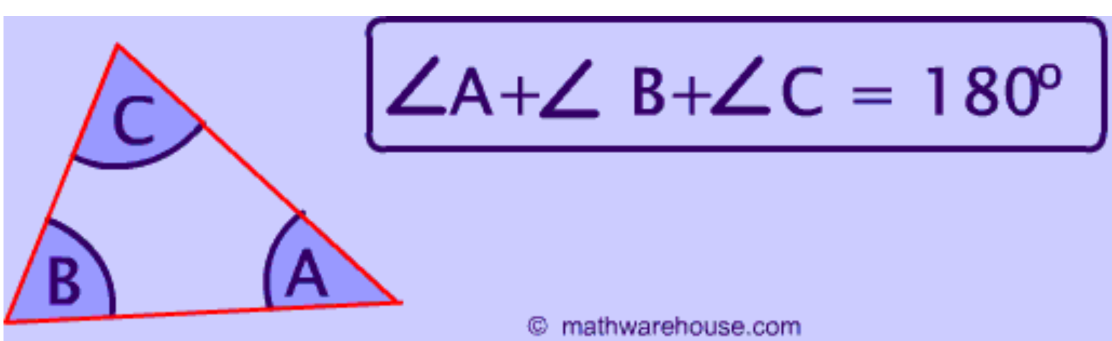
4. Guided Practice. Use the diagram to find $m\measuredangle MJK$.



A **corollary** is a theorem whose proof follows directly from another theorem. Here are two corollaries to the Triangle Sum Theorem.

Corollaries

COROLLARY		HYPOTHESIS	CONCLUSION
4-2-2	The acute angles of a right triangle are complementary.		$\angle D$ and $\angle E$ are complementary. $m\angle D + m\angle E = 90°$
4-2-3	The measure of each angle of an equiangular triangle is 60°.		$m\angle A = m\angle B = m\angle C = 60°$



<u>Video Example 2</u>. One of the acute angles of a right triangle measures 15.3°. What is the measure of the other acute angle?

2 Finding Angle Measures in Right Triangles

One of the acute angles in a right triangle measures 22.9°. What is the measure of the other acute angle?

Let the acute angles be $\angle M$ and $\angle N$, with $m\angle M = 22.9°$.

$m\angle M + m\angle N = 90$	*Acute ∠s of rt. △ are comp.*
$22.9 + m\angle N = 90$	*Substitute 22.9 for m∠M.*
$m\angle N = 67.1°$	*Subtract 22.9 from both sides.*

<u>Example 2</u>. One of the acute angles in a right triangle measures $2x°$. What is the measure of the other acute angle?

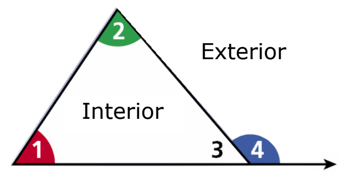
<u>Guided Practice</u>.

5. The measure of one of the acute angles in a right triangle is 63.7°. What is the measure of the other acute angle?

6. The measure of one of the acute angles in a right triangle is $x°$. What is the measure of the other acute angle?

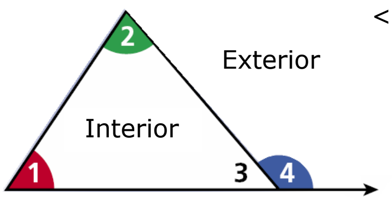
7. The measure of one of the acute angles in a right triangle is $48\frac{2}{3}°$. What is the measure of the other acute angle?

The **interior** is the set of all points inside the figure. The **exterior** is the set of all points outside the figure.



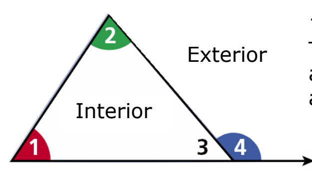
An **interior angle** is formed by two sides of a triangle. An **exterior angle** is formed by one side of the triangle and extension of an adjacent side.

<4 is an exterior angle.



<3 is an interior angle.

Each exterior angle has two remote interior angles. A **remote interior angle** is an interior angle that is not adjacent to the exterior angle.



<4 is an exterior angle.

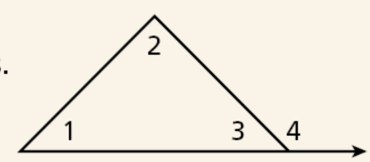
The remote interior angles of <4 are <1 and <2.

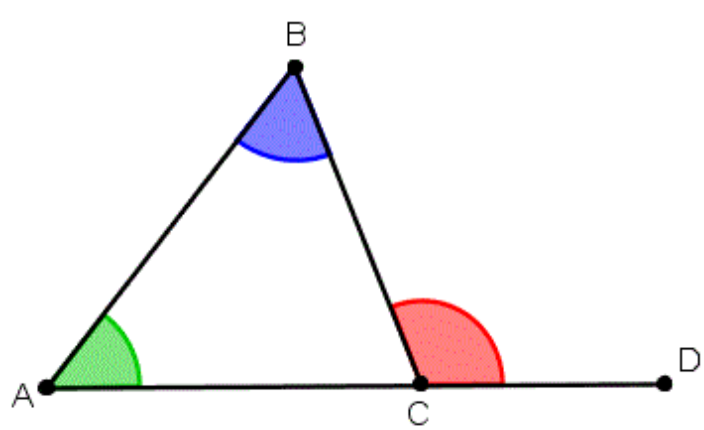
<3 is an interior angle.

Theorem 4-2-4 **Exterior Angle Theorem**

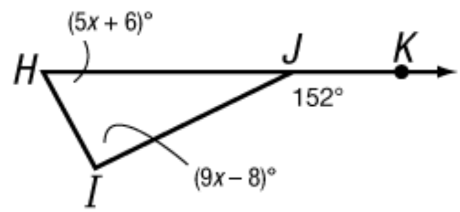
The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

$m\angle 4 = m\angle 1 + m\angle 2$

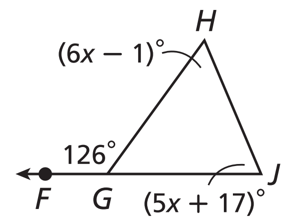




<u>Video Example 3</u>. Find $m\measuredangle H$.



3 Applying the Exterior Angle Theorem



Find m∠J.

$m\angle J + m\angle H = m\angle FGH$ — *Ext. ∠ Thm.*

$5x + 17 + 6x - 1 = 126$ — *Substitute 5x + 17 for m∠J, 6x – 1 for m∠H, and 126 for m∠FGH.*

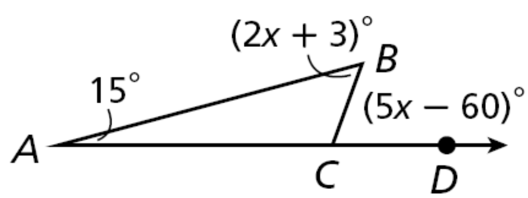
$11x + 16 = 126$ — *Simplify.*

$11x = 110$ — *Subtract 16 from both sides.*

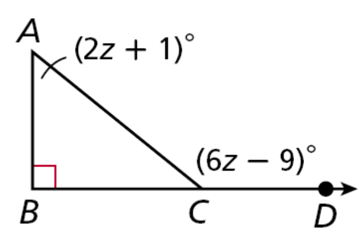
$x = 10$ — *Divide both sides by 11.*

$m\angle J = 5x + 17 = 5(10) + 17 = 67°$

<u>Example 3</u>. Find $m\measuredangle B$.

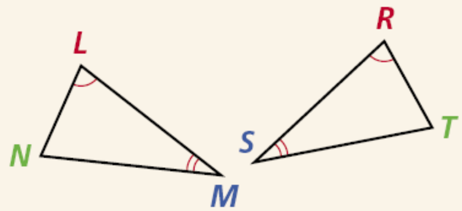


8. <u>Guided Practice</u>. Find $m\measuredangle ACD$.

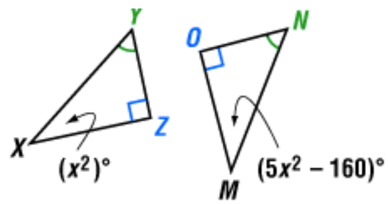


<u>4-3 Angle Relationships in Triangles</u> *(p 236)* 15, 17-20.

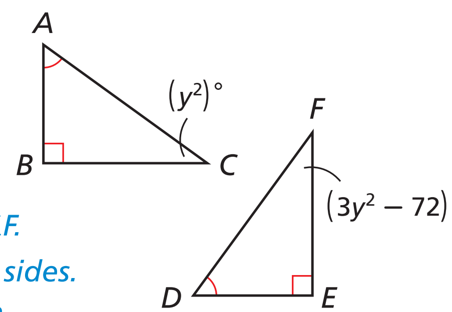
Theorem 4-2-5 Third Angles Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.		$\angle N \cong \angle T$

<u>Video Example 4</u>. Find $m\measuredangle X$ & $m\measuredangle M$.



4 Applying the Third Angles Theorem



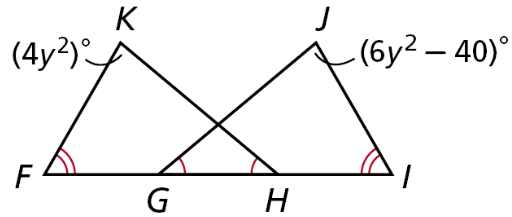
Find m∠C and m∠F.

$\angle C \cong \angle F$	*Third ∆s Thm.*
$m\angle C = m\angle F$	*Def. of ≅ ∆s.*
$y^2 = 3y^2 - 72$	*Substitute y^2 for m∠C and $3y^2 - 72$ for m∠F.*
$-2y^2 = -72$	*Subtract $3y^2$ from both sides.*
$y^2 = 36$	*Divide both sides by −2.*

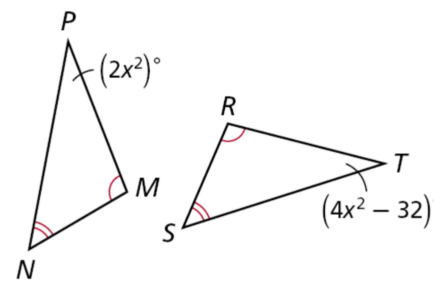
So $m\angle C = 36°$.

Since $m\angle F = m\angle C$, $m\angle F = 36°$.

Example 4. Find $m\measuredangle K$ & $m\measuredangle J$.



9. Guided Practice. Find $m\measuredangle P$ & $m\measuredangle T$.



4-3 Angle Relationships in Triangles *(p 236)* 15, 17-22, 24, 29, 33, 38, 39.